AN ELECTROMAGNETIC ANALYSIS OF THE TRANSMISSION PROPERTIES OF RADIAL OFFSETS IN ROUND OPTICAL FIBERS

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ABSTRACT

The transmission loss of a radial offset in a round optical fiber is computed by a rigorous electromagnetic approach. The e.m. field in both the incoming and outgoing fibers is developed into a sum of modes and the power transmission properties of the offset are established on a mode-to-mode basis. This gives an extremely accurate and complete picture of the physical phenomenon, whose validity is confirmed by the very close agreement with available experimental data. The theory can also account for the reflected power which may be non-negligible for fibers with a large numerical aperture.

Introduction

The radial misalignment of the fiber axes is known to to be the most severe source of coupling losses in splices between round optical fibers[1-2]. Thus the problem of accurately characterizing the transmission properties of a radial offset is a rather fundamental one in view of the design of optical communication links including a considerable number of splices.
To solve this problem, a number of approximate methods, mostly relying upon geometric optics[1-4], have been presented in the past. Generally speaking, the most important limitations of the above consist in that they are usually not able to accurately describe the dependence of coupling losses on fiber axis displacement for very small offsets and for offsets comparable with or exceeding the core diameter.
As an alternative to the geometrical approach, a fully electromagnetic solution based on the numerical description of the fields propagating in the fiber is presented in this paper. According to this method, the electromagnetic field in the incoming fiber is represented by a summation extended to all modes, both incident and reflected. The field at the discontinuity plane is then expanded in terms of the modes of the outgoing fiber which allows the power transmission properties of the radial offset to be described on a mode-to-mode basis. As a result, a truly accurate and complete characterization of the e.m. behaviour of the discontinuity is obtained, yielding excellent agreement with available experimental data. The topics covered include the dependence of the splice loss on fiber axis displacement and mode power distribution in the incoming fiber, both for single and multimode operation.

Theoretical formulation

Let us consider an imperfect splice between two identical clad optical fibers, involving a radial offset d between the fiber axes but negligible end-face separation and tilt. It is the purpose of this section to derive the power scattering properties of the offset discontinuity, that we will assume to occur at the plane z=0. All of the physical quantities referring to the incoming (outgoing) fiber will be denoted by the subscript 1(2).

The transverse components of the electromagnetic field in the incoming fiber at the discontinuity plane may be be represented as:

$$\begin{cases} \vec{E}_{in} = \sum_i c_i^+ \vec{E}_{1i} + \sum_i c_i^- \vec{E}_{1i} \\ \vec{H}_{in} = \sum_i c_i^+ \vec{H}_{1i} - \sum_i c_i^- \vec{H}_{1i} \end{cases} \qquad (1)$$

where c_i^+, c_i^- are the complex amplitudes of the incident and reflected i-th mode, respectively. It is assumed that the mode functions $(\vec{E}_{1i}, \vec{H}_{1i})$ are normalized in such a way that:

$$\int_{S_1} \vec{E}_{1i} \times \vec{H}_{1i}^{*} \bullet \hat{z}\, dS = 1 \qquad (2)$$

where S_1 is the fiber cross section and $\hat{z}$ denotes the unit vector in the direction of propagation. Generally speaking, leaky modes could be included in the summation (1) to account for the radiation field of the incoming fiber, but this is usually unnecessary for practical applications.
The transmitted field in the outgoing fiber will be represented by a superposition of guided and radiation modes. Thus the boundary conditions at the discontinuity plane may be written as:

$$\sum_i c_i^+ \vec{E}_{1i} + \sum_i c_i^- \vec{E}_{1i} = \sum_j t_j \vec{E}_{2j} + \int_\beta T(\beta) \vec{E}_{2R}(\beta) d\beta \qquad (3)$$

$$\sum_i c_i^+ \vec{H}_{1i} - \sum_i c_i^- \vec{H}_{1i} = \sum_j t_j \vec{H}_{2j} + \int_\beta T(\beta) \vec{H}_{2R}(\beta) d\beta \qquad (4)$$

where the t_j's and $T(\beta)$ are unknown transmission coefficients. Note that the mode functions $(\vec{E}_{2i}, \vec{H}_{2i})$ are the same as for the guided modes of the incoming fiber, except for a shift in the transverse plane.
To find the transmission coefficients for the guided modes we now cross-multiply the two sides of (3) by $\vec{H}_{2j}^{*}$ and the two sides of (4) by $\vec{E}_{2j}^{*}$ (* denotes the complex conjugate) and make use of the orthogonality properties of the fiber modes to obtain:

$$\begin{cases} t_j = \sum_i T'_{ji} (c_i^+ + c_i^-) \\ t_j = \sum_i T''_{ji} (c_i^+ - c_i^-) \end{cases} \qquad (5)$$

with

$$T'_{ji} = \int_{S_2} \vec{E}_{1i} \times \vec{H}_{2j}^{*} \bullet \hat{z}\, dS \qquad (6)$$

$$T''_{ji} = \int_{S_2} \vec{E}_{2j}^{*} \times \vec{H}_{1i} \bullet \hat{z}\, dS\,. \qquad (7)$$

For convenience, (5) may be rearranged in matrix form as

$$\begin{cases} \underline{t} = \underline{T}'(\underline{c}^+ + \underline{c}^-) \\ \underline{t} = \underline{T}''(\underline{c}^+ + \underline{c}^-) \end{cases} \qquad (8)$$

where the meaning of the vectors $\underline{t}$, $\underline{c}^+$, $\underline{c}^-$ is obvious and $\underline{T}'$, $\underline{T}''$ are square matrices whose elements are given by (6), (7). Solving (8) yields

$$\begin{cases} \underline{c}^- = \boldsymbol{\rho}\, \underline{c}^+ \\ \underline{t} = \underline{\tau}\, \underline{c}^+ \end{cases} \qquad (9)$$

with

$$\begin{cases} \underline{\rho} = (\underline{I}'' + \underline{I}')^{-1} (\underline{I}'' - \underline{I}') \\ \underline{\tau} = \underline{I}'(\underline{I} + \underline{\rho}) \qquad (\underline{I}\text{=identity matrix}), \end{cases} \tag{10}$$

In principle ,(9) and (10), together with the definitions (6) and (7), provide a complete solution to the offset problem. However, in all practical cases the main interest is focused on the effects of the discontinuity on optical power flow and distribution within the fiber. From this point of view,(9) turn out to be unnecessarily complicated,since they involve the phases of the guided modes,which usually can neither be predicted nor measured.

To restrict the analytical description to the scattering of active power,we now make the assumption that the phases of the complex amplitudes c_i^+ be random uncorrelated quantities,i.e.,

$$< c_i^+ (c_j^+)^* > = 0 \tag{11}$$

for $i \neq j$, where < >denotes the statistical average[7]. Thus any of eqs. (9), say

$$t_j = \sum_i \tau_{ji} c_i^+ \tag{12}$$

can be transformed into a power equation of the form

$$<|t_j|^2> = \sum_i |\tau_{ji}|^2 <|c_i^+|^2> \tag{13}$$

In turn,(13) suggests the introduction of a *power reflection matrix*, $\underline{R}$,and a *power transmission matrix*, $\underline{T}$ that are defined as

$$\begin{aligned} \underline{R} &= [\ |\rho_{ji}|^2\ ] \\ \underline{T} &= [\ |\tau_{ji}|^2\ ]. \end{aligned} \tag{14}$$

Finally, if the vectors of the average power carried by the incident, reflected and transmitted modes are introduced and denoted by $\underline{P}^+$,$\underline{P}^-$ and $\underline{P}_T (P_i = <|c_i^+|^2>$,etc) from (9),(13) we get

$$\begin{cases} \underline{P}^- = \underline{R}\ \underline{P}^+ \\ \underline{P}_T = \underline{T}\ \underline{P}^+ \end{cases} \tag{15}$$

which is the required description of the active power scattering from the discontinuity.

The only data required to numerically evaluate $\underline{R}$ and $\underline{T}$ through (14) , (10) and (6,7) is the refractive-index distribution in the fiber core:all of the calculations may then be carried out automatically with the aid of a general-purpose package such as described in[6]. Once this has been done, any desired information can be obtained from (15). For instance, the power coupling efficiency will be given by

$$\eta = \frac{\sum_j P_{Tj}}{\sum_i P_i^+} = \frac{\sum\sum_{ji} T_{ji} P_i^+}{\sum_i P_i^+} \tag{16}$$

and will thus be a function of the power distribution in the incoming fiber (i.e., the P_i^+ values).

Numerical results

Based on the method outlined in the previous section, extensive calculations were carried out both for step- and graded-index optical fibers. For the sake of brevity we will only report here a few selected results allowing a direct comparison with experimental data available in the literature. The cases considered are a single-mode and a multimode step-index optical fiber for which the transmission properties of the offset discontinuity were extensively charactarized by Bisbee[8]. The relevant physical and geometrical parameters are as follows:

Core refractive index	n_1=1.6171
Cladding refractive index	n_2=1.6127 (single-mode fiber)
Cladding refractive index	n_2=1.6038 (multimode fiber)
Core diameter	2a=3.7μm (single-mode fiber)
Core diameter	2a=10.8μm (multimode fiber)
Wavelength	λ=0.6328μm

For the single-mode fiber case the power coupling efficiency as defined by (16) is plotted in fig.1 (solid line) against the percentage offset,and is compared with Bisbee's experimental results (triangles).

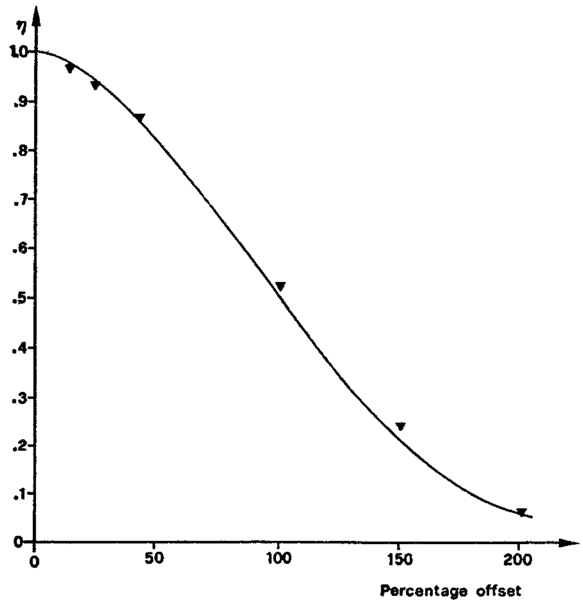

Fig. 1

The agreement between the two sets of values is clearly good. Since no data are given in [8]for zero end separation, to draw this figure the data measured by Bisbee for an end separation of 5.08μm with index matching oil were normalized to yield a 100% efficiency at zero offset. This may account for the slight discrepancies between theoretical and measured results, as is evident by inspection of fig.7 in[8] .

The case of a multimode fiber is considerably more interesting since now the power transmission properties of the offset are affected by the power distribution among the guided modes of the incoming fiber. This is illustrated in fig.2, where the power coupling efficiency versus percentage offset is presented for: a)uniform excitation of all guided modes and b) steady-state distribution.The latter is defined according to[9] as

$$P_i^+ = J_0\left(2.405 \cdot \sqrt{\frac{K^2 n_1^2 - \beta_i^2}{K^2 n_1^2 - K^2 n_2^2}}\right), \tag{17}$$

where β_i is the phase constant of the i-th mode, K represents the free-space wavenumber and J_0 is the 0-th order Bessel function. The experimental points measured by Bisbee are also reported in the figure and are seen to provide excellent agreement with the computations for the steady-state distribution.

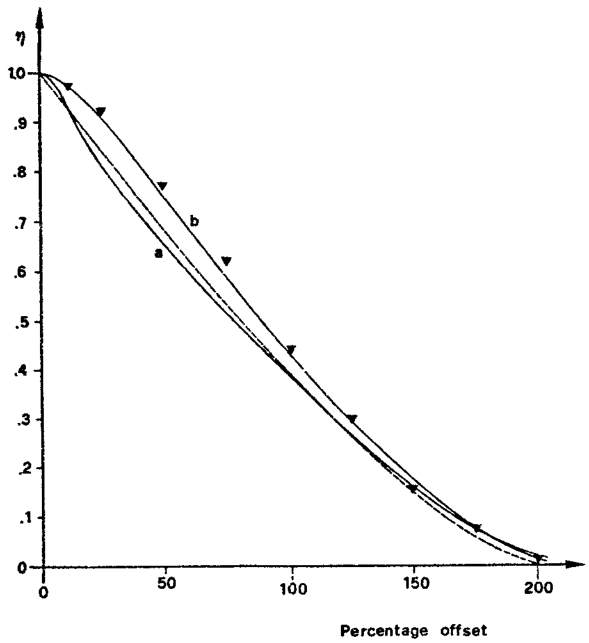

Fig. 2

The case of uniform excitation yields considerably larger transmission losses especially for small offsets, in agreement with Gloge's results[3]. Note that (17) can be considered a reasonable guess to the actual power distribution in the incoming fiber for Bisbee's experiment because of the combined effects of the Gaussian-beam excitation, the length of the fiber itself and the mode-strippers[8]. The error involved in (17) is probably responsible for the small discrepancies between computed and measured results in fig. 2.
The dashed line plotted in the same figure represents the coupling efficiency as evaluated by geometric optics for uniform excitation. This quantity is analytically expressed as [5-10-11]:

$$\eta = \frac{2}{\pi} \arccos\frac{d}{2a} - \frac{d}{\pi a} \cdot \sqrt{1 - \left(\frac{d}{2a}\right)^2} \quad . \qquad (18)$$

As shown in the figure, the results of the computer calculations for the case of uniform power distribution are reasonably well approximated, on the average, by (18), except for the range of very large offsets. In fact, (18) yields $\eta=0$ for $d=2a$ and becomes meaningless for $d > 2a$, while both the computed and measured efficiency are always finite and smoothly tend to 0 as d/a tends to infinity.
The behaviour of the coupling efficiency for small offset values is more clearly displayed in fig. 3, where the upper left section of fig. 2 is repeated with expanded scales.
Note that the computer approach yields a sensitivity to small offsets definitely less than predicted by ray optics, in agreement with a number of experimental observations (e.g.[4]). In particular a zero slope is found at $d=0$, no matter what the power distribution. This figure further shows how the detailed aspects of the physical problem can be mastered by the electromagnetic solution proposed in this paper to a much greater extent than by previous approximate methods.

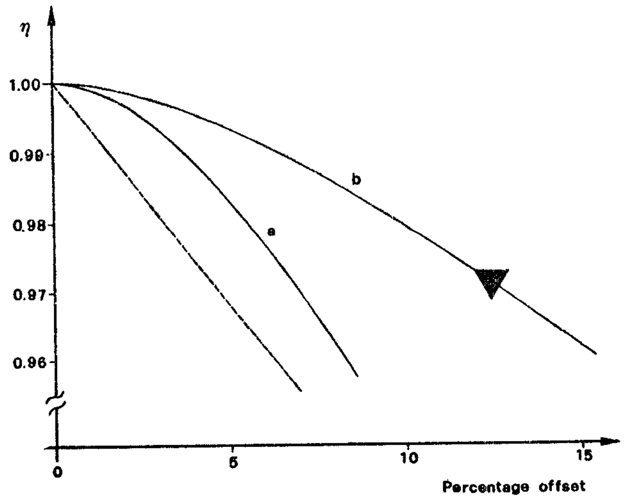

Fig. 3

Acknowledgement

This work was partially sponsored by the Italian National Research Council (CNR).

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